Hong Kong Mathematics Olympiad (2007 – 2008)

Final Event 1 (Individual)

香港數學競賽 (2007 – 2008)

決賽項目 1 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 設直線 $A=15\times\tan 44°\times\tan 45°\times\tan 46°$ ，求 A 的值。

Let $A=15\times\tan 44°\times\tan 45°\times\tan 46°$, find the value of A.

2. 設 n 為正整數及 $\overbrace{20082008\cdots 200815}^{n\text{ 個 }2008}$ 能被 A 整除。若 n 的最小可能值是 B，求 B 的值。

Let n be positive integer and $\overbrace{20082008\cdots 200815}^{n\text{ 2008's}}$ is divisible by A. If the least possible value of n is B, find the value of B.

3. 已知有 C 個整數滿足方程 $|x-2|+|x+1|=B$ ，求 C 的值。

Given that there are C integers that satisfy the equation $|x-2|+|x+1|=B$, find the value of C.

4. 在坐標平面上，點 $(-C, 0)$ 與直線 $y=x$ 的距離是 $\sqrt{D}$ ，求 D 的值。

In the coordinate plane, the distance from the point $(-C, 0)$ to the straight line $y=x$ is $\sqrt{D}$, find the value of D.

Hong Kong Mathematics Olympiad (2007 – 2008)

Final Event 2 (Individual)

香港數學競賽 (2007 – 2008)

決賽項目 2 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 設 $\sqrt{a}=\sqrt{7+\sqrt{13}}-\sqrt{7-\sqrt{13}}$，求 P 的值。

Given that $P=\left[\sqrt[3]{6}\times\left(\sqrt[3]{\frac{1}{162}}\right)\right]^{-1}$, find the value of P.

2. 設 a、b 和 c 是實數且 $b:(a+c)=1:2$ 及 $a:(b+c)=1:P$。若 $Q=\frac{a+b+c}{a}$，求 Q 的值。

Let a, b and c be real numbers with ratios $b:(a+c)=1:2$ and $a:(b+c)=1:P$. If $Q=\frac{a+b+c}{a}$, find the value of Q.

3. 設 $R=\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)^{Q}+\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)^{Q}$，求 R 的值。

Let $R=\left(\sqrt{\sqrt{3}+\sqrt{2}}\right)^{Q}+\left(\sqrt{\sqrt{3}-\sqrt{2}}\right)^{Q}$, find the value of R.

4. 設 $S=(x-R)^2+(x+5)^2$，其中 x 為實數，求 s 的最小值。

Let $S=(x-R)^2+(x+5)^2$, where x is a real number. Find the minimum value of S.

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 3 (Individual)
香港數學競賽 (2007 – 2008)
決賽項目 3 (個人)

除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 $\frac{1-\sqrt{3}}{2}$ 滿足方程 $x^2+px+q=0$，其中 p 和 q 是有理數。若 $A=|p|+2|q|$，求 A 的值。

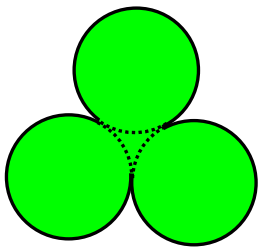
Given that $\frac{1-\sqrt{3}}{2}$ satisfies the equation $x^2+px+q=0$, where p and q are rational numbers. If $A=|p|+2|q|$, find the value of A.

2. U_1 及 U_2 兩代有大小相同的紅球和白球。U_1 裝有 A 個紅球，2 個白球。U_2 裝有 2 個紅球，B 個白球。從每袋中各取出兩個球。若取到四個紅球的概率是 $\frac{1}{60}$，求 B 的值。

Two bags U_1 and U_2 contain identical red and white balls. U_1 contains A red balls and 2 white balls. U_2 contains 2 red balls and B white balls. Take two balls out of each bag. If the probability of all four balls are red is $\frac{1}{60}$, find the value of B.

3. 圖一由三個大小相同且互切的圓所組成，三個圓的半徑均是 B cm。若陰影部分的周界是 C cm，求 C 的值。(取 $\pi=3$)

Figure 1 is formed by three identical circles touching one another, the radius of each circle is B cm. If the perimeter of the shaded region is C cm, find the value of C. (Take $\pi=3$)



圖一
Figure 1

4. 設與 $\sqrt{C}$ 最接近的整數是 D，求 D 的值。

Let D be the integer closet to $\sqrt{C}$, find the value of D .

Hong Kong Mathematics Olympiad (2007 – 2008)
Final Event 4 (Individual)
香港數學競賽 (2007 – 2008)
決賽項目 4 (個人)

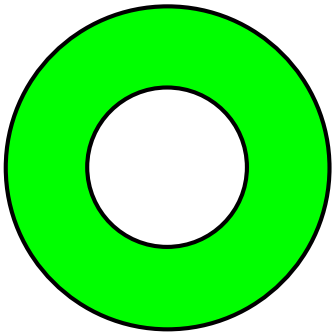
除非特別聲明，答案須用數字表達，並化至最簡。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知 x 及 y 為實數，且滿足 $|x|+x+y=10$ 及 $|y|+x-y=10$。若 $P=x+y$，求 P 的值。

Given that x and y are real numbers such that $|x|+x+y=10$ and $|y|+x-y=10$. If $P=x+y$, find the value of P.

2. 如圖一，陰影部分由兩同心圓所組成，其面積為 96π cm^2。若該兩圓的半徑相差 $2P$ cm 及大圓的面積為 Q cm^2，求 Q 的值。

In Figure 1, the shaded area is formed by two concentric circles and has area 96π cm^2. If the two radii differ by $2P$ cm and the large circle has area Q cm^2, find the value of Q. (Take $\pi=3$)



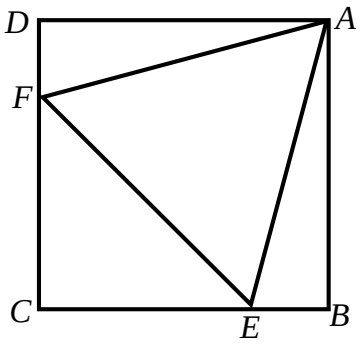
圖一
Figure 1

3. 設 R 為最大的整數使得 $R^Q<5^{200}$，求 R 的值。

Let R be the largest integer such that $R^Q<5^{200}$, find the value of R.

4. 圖二顯示一邊長為 $(R-1)$ cm 的正方形 $ABCD$ 及一等邊三角形 AEF (E 及 F 分別是直線 BC 及 CD 上的點)。若 ΔAEF 的面積是 $(S-3)$ cm^2，求 S 的值。

In Figure 2 , there are a square $ABCD$ with side length $(R-1)$ cm and an equilateral triangle AEF (E and F are points on BC and CD respectively). If the area of ΔAEF is $(S-3)$ cm^2 , find the value of S .



圖二

Figure 2